Math 101 Chapter 4/Section: 3 Topic: Saving for the Long Term, Worksheet

Finish the formulas for the following in terms of:

t = number of deposits, y = years, r = monthly interest rate, APR = annual percentage rate

1. Balance after t monthly deposits =

2. Needed monthly deposit =

3. Monthly annuity yield =

4. Nest egg needed =

Match the following questions to the equations (1-4) above:

_____ How much money would you need to deposit each month in order to reach your desired result?

_____ If you start with money in the bank and withdraw the same amount of money for each month, how much would you be able to withdraw each month?

_____ How much money you would have after depositing money for t months?

_____ If you know how much money you want to withdraw each month for 't' months, how much do you have to start with

Solve the following problems:

1. You open a savings account and deposit $200 into it at the end of each month. The account pays you a monthly interest rate of 1.5% on the balance in the account at the beginning of each month. At the end of the first month the balance is $200. At the end of the second month the balance is $403. Track the growth of this account through 6 months. (***Hint:** New balance = Previous balance + Interest + Deposit. You start with $0 in your savings account.)

2. Suppose we have a savings account earning 8% APR. We deposit $30 into the account at the end of each month for 4 years. What is the account balance after 5 years?

3. How much do you need to deposit each month into your savings account that has an APR of 9% in order to have $30,000 for your college education in 3 years?

4. Suppose we have $1,000,000 in the bank with an APR of 6.3% compounded monthly. Find the monthly yield for a 30-year annuity.

5. Suppose your retirement account pays 4.9% APR compounded monthly. What size nest egg do you need in order to retire with a 20-year annuity that yields $5,000 a month?

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ANSWER KEY:

Finish the formulas for the following in terms of:

t = number of deposits, y = years, r = monthly interest rate, APR = annual percentage rate

*** Right Column:** This is when the rate (r) is given in the problem as a yearly rate (APR). The yearly rate (APR) must be divided into 12 months because the problem is asking about monthly deposits.

1. Balance after t monthly deposits =

$$\frac{\text{Deposit} \times \left((1+r)^{t}-1\right)}{r} \quad \text{OR} \quad \frac{\text{Deposit} \times \left(\left(1+\frac{\text{APR}}{12}\right)^{(12y)}-1\right)}{\left(\frac{\text{APR}}{12}\right)}$$

2. Needed monthly deposit =

$$\frac{\text{Goal} \times r}{\left((1+r)^{t}-1\right)} \quad \text{OR} \quad \frac{\text{Goal} \times \left(\frac{\text{APR}}{12}\right)}{\left(\left(1+\frac{\text{APR}}{12}\right)^{(12y)}-1\right)}$$

3. Monthly annuity yield =

$$\frac{\text{Nest egg} \times r \times (1+r)^{t}}{\left((1+r)^{t}-1\right)} \quad \text{OR} \quad \frac{\text{Nest egg} \times \frac{\text{APR}}{12} \times \left(1+\frac{\text{APR}}{12}\right)^{(12y)}}{\left(\left(1+\frac{\text{APR}}{12}\right)^{(12y)}-1\right)}$$

ANSWER KEY

4. Nest egg needed =

$$\frac{\text{Monthly annuity yield} \times \left((1+r)^{t}-1\right)}{\left(r \times (1+r)^{t}\right)} \quad \text{OR} \quad \frac{\text{Monthly annuity yield} \times \left(\left(1+\frac{\text{APR}}{12}\right)^{(12y)}-1\right)}{\left(\frac{\text{APR}}{12} \times \left(1+\frac{\text{APR}}{12}\right)^{(12y)}\right)}$$

Match the following questions to the equations (1-4) above:

__2__ How much money would you need to deposit each month in order to reach your desired result?

__3__ If you start with money in the bank and withdraw the same amount of money for each month, how much would you be able to withdraw each month?

__1__ How much money you would have after depositing money for t months?

__4__ If you know how much money you want to withdraw each month for 't' months, how much do you have to start with?

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ANSWER KEY

Solve the following problems:

1. You open a savings account and deposit $200 into it at the end of each month. The account pays you a monthly interest rate of 1.5% on the balance in the account at the beginning of each month. At the end of the first month the balance is $200. At the end of the second month the balance is $403. Track the growth of this account through 6 months. (***Hint:** New balance = Previous balance + Interest + Deposit. You start with $0 in your savings account.)

Use Formula:

New balance = Previous balance + Interest + Deposit

Balance after 1 deposit = $0 + ($0 x 0.015) + $200 = $200

Balance after 2 deposits = $200 + ($200 x 0.015) + $200 = $403

Balance after 3 deposits = $403 + ($403 x 0.015) + $200 = $609.05

…and so on

At end of month #	Interest paid on previous balance	Deposit	New balance
1	$0	$200	**$200**
2	1.5% of $200	$200	**$403**
3	1.5% of $403	$200	**$609.05**
4	1.5% of $609.05	$200	**$818.19**
5	1.5% of $818.18	$200	**$1030.46**
6	1.5% of $1030.45	$200	**$1245.92**

ANSWER KEY

2. Suppose we have a savings account earning 8% APR. We deposit $30 into the account at the end of each month for 4 years. What is the account balance after 4 years?

Use formula:

Balance after t monthly deposits =

$$\frac{\text{Deposit} \times \left(\left(1+\frac{\text{APR}}{12} \right)^{(12y)} - 1 \right)}{\left(\frac{\text{APR}}{12} \right)}$$

= [$30 x ([1+0.08/12]^(12 x 4) - 1)] / [0.08/12]

= 1690.497

The balance after 4 years is **$1690.50**

3. How much do you need to deposit each month into your savings account that has an APR of 9% in order to have $30,000 for your college education in 3 years?

Use formula:

Needed monthly deposit =

$$\frac{\text{Goal} \times \left(\frac{\text{APR}}{12} \right)}{\left(\left(1+\frac{\text{APR}}{12} \right)^{(12y)} - 1 \right)}$$

= [$30,000 x (0.09/12)] / [(1+[0.09/12])^(12 x 3) - 1]

= $728.992

Round up because if you round down you will not reach your goal.

You need to deposit **$729.00** monthly

ANSWER KEY

4. Suppose we have $1,000,000 in the bank with an APR of 6.3% compounded monthly. Find the monthly yield for a 30-year annuity.

Use formula:

Monthly annuity yield =

$$\frac{\text{Nest egg} \times \frac{\text{APR}}{12} \times \left(1+\frac{\text{APR}}{12}\right)^{(12y)}}{\left(\left(1+\frac{\text{APR}}{12}\right)^{(12y)}-1\right)}$$

= [$1,000,000 x (0.063/12) x (1+0.063/12)^(12 x 30)] / [(1+0.063/12)^(12 x 30) - 1]

=$6189.728

The monthly annuity yield is **$6189.73**

5. Suppose your retirement account pays 4.9% APR compounded monthly. How much do you need in order to retire with a 20-year annuity that yields $5,000 a month?

Use formula:

Nest egg needed =

$$\frac{\text{Annuity yield goal} \times \left(\left(1+\frac{\text{APR}}{12}\right)^{(12y)}-1\right)}{\left(\frac{\text{APR}}{12} \times \left(1+\frac{\text{APR}}{12}\right)^{(12y)}\right)}$$

= [$5,000 x ([1+0.049/12]^(12 x 20) - 1)] / [(0.049/12) x (1+0.049/12)^(12 x 20)]

= $764,007.253

Round up because if you round down you will not reach your goal.

The nest egg needed is **$764,007.26**

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